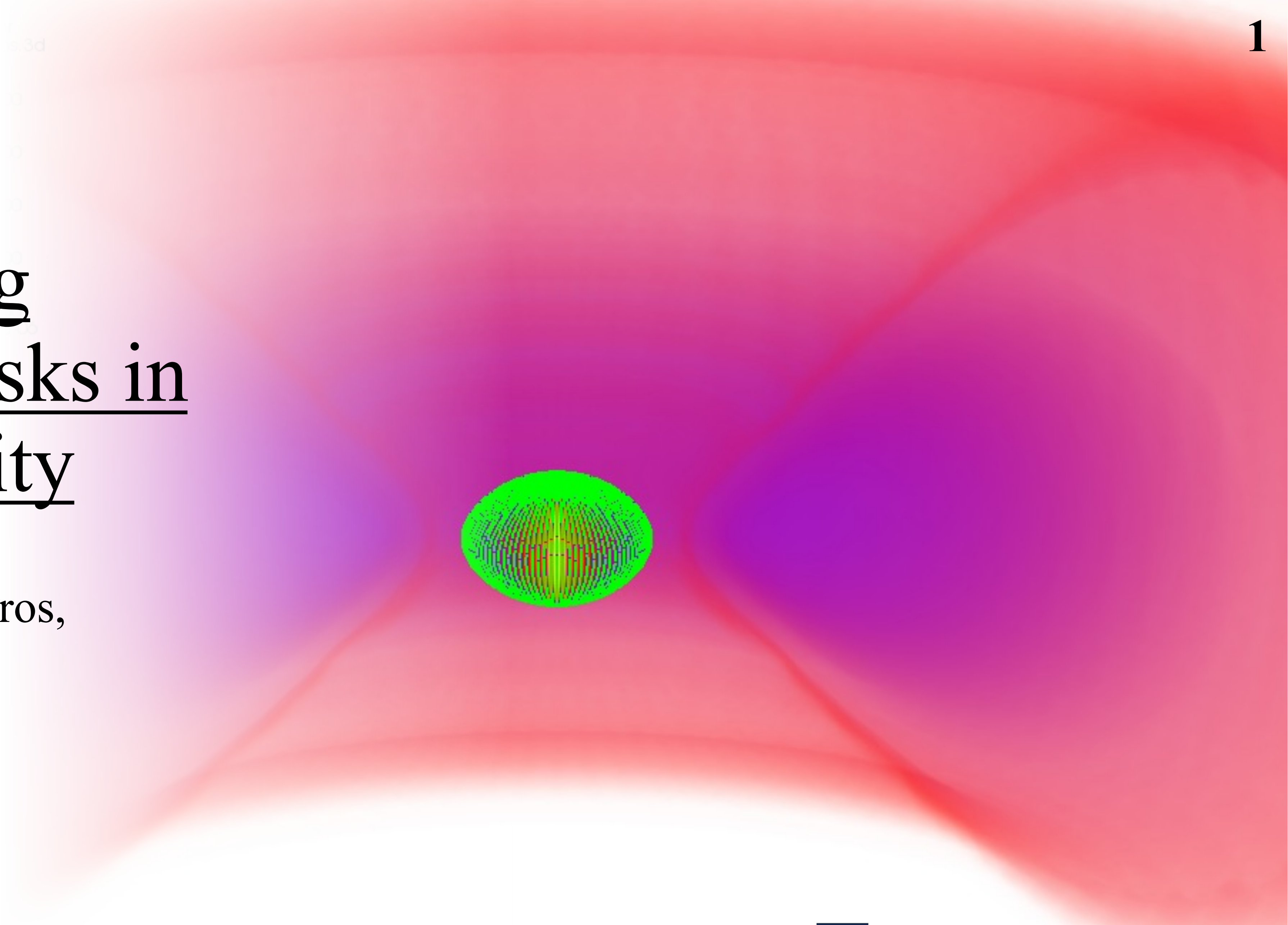
Self-gravitating neutron star-disks in general relativity



Eric Yu, Antonios Tsokaros, Milton Ruiz, Kōji Uryū

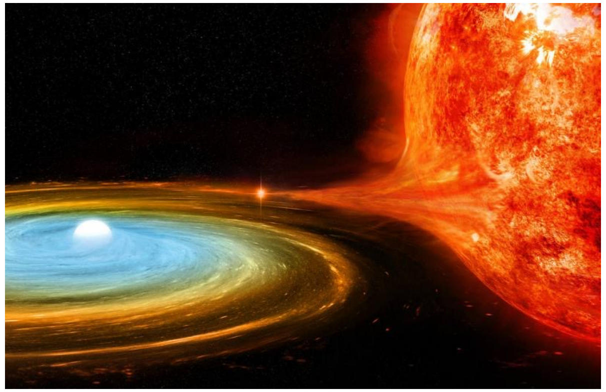
University of Illinois at Urbana Champaign

ericyu3@illinois.edu

ILLINOIS

Why we study neutron star-disks (NSDs)?

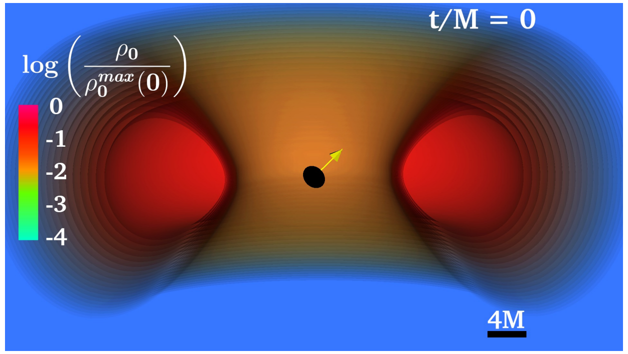
- NSNS merger leads to BH-disk (GW170817?) or NS-disk
- X-ray binary where accretor is an NS
- Collapse of supermassive star into an NS-disk

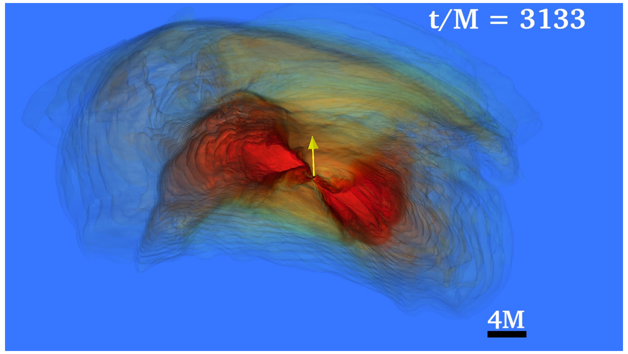


Credit: NASA/CXC/M. Weiss

Why is disk self-gravity important?

- Eccentric NSNS mergers result in disk up to 10% mass of initial (Gold 2012)
- Certain mass ratios, NS spins, and NS EOS result in large disk mass (Krüger 2020)
- Precession of angular momenta if compact object and disk are misaligned (Tsokaros 2022)
- Gravitational waves from disk instabilities (Wessel 2023)





Credit: Eric Yu, Mit Kotak,

How do we compute NSDs?

General relativity

4 dimensional spacetime

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

$$dx^\alpha = (dt, dx, dy, dz)$$

$$G_{\alpha\beta} = \frac{8\pi G}{c^4} T_{\alpha\beta}$$

Numerical relativity

3+1 decomposition into space + time

$$ds^2 = -\alpha^2 dt^2 + \psi^4 \tilde{\gamma}_{ab}(dx^a + \beta^b dt)(dx^b + \beta^b dt)$$

$$dx^a = (dx, dy, dz)$$

$$\mathring{\Delta}\psi = \mathcal{S}_{\mathrm{H}}, \mathring{\Delta}\tilde{\beta}_a = \mathcal{S}_a, \mathring{\Delta}(\alpha\psi) = \mathcal{S}_{\mathrm{tr}}, \mathring{\Delta} h_{ab} = \mathcal{S}_{ab}$$

$$\tilde{\gamma}_{ab} = f_{ab} + h_{ab}, \tilde{\beta}_a = \tilde{\gamma}_{ab}\beta^b$$

How do we compute NSDs?

$$\mathcal{S}_{\mathrm{H}} := -h^{ab}\mathring{D}_a\mathring{D}_b\psi + \tilde{\gamma}^{ab}C^c_{ab}\mathring{D}_c\psi + \frac{\psi}{8}{}^3\tilde{R} - \frac{\psi^5}{8}\left(\tilde{A}_{ab}\tilde{A}^{ab} - \frac{2}{3}K^2\right) - 2\pi\psi^5\rho_{\mathrm{H}},$$

$$\mathcal{S}_a := -h^{bc}\mathring{D}_b\mathring{D}_c\tilde{\beta}_a + \tilde{\gamma}^{bc}\mathring{D}_b(C^d_{ca}\tilde{\beta}_d) + \tilde{\gamma}^{bc}C^d_{bc}\tilde{D}_d\tilde{\beta}_a + \tilde{\gamma}^{bc}C^d_{ba}\tilde{D}_c\tilde{\beta}_d - \frac{1}{3}\mathring{D}_a(h^{bc}\mathring{D}_b\tilde{\beta}_c - \tilde{\gamma}^{bc}C^d_{bc}\tilde{\beta}_d) - \frac{1}{3}\mathring{D}_a\mathring{D}^b\tilde{\beta}_b - {}^3\tilde{R}_{ab}\tilde{\beta}^b - 2\alpha\tilde{A}_a{}^b\frac{\alpha}{\psi^6}\tilde{D}_b\left(\frac{\psi^6}{\alpha}\right) + \frac{4}{3}\alpha\tilde{D}_aK + 16\pi\alpha j_a,$$

$$\mathcal{S}_{\mathrm{tr}} := -h^{ab}\mathring{D}_a\mathring{D}_b(\alpha\psi) + \tilde{\gamma}^{ab}C^c_{ab}\mathring{D}_c(\alpha\psi) + \frac{\alpha\psi}{8}{}^3\tilde{R} + \psi^5\pounds_\omega K + \alpha\psi^5\left(\frac{7}{8}\tilde{A}_{ab}\tilde{A}^{ab} + \frac{5}{12}K^2\right) + 2\pi\alpha\psi^5(\rho_{\mathrm{H}} + 2S),$$

$$\mathcal{S}_{ab} := 2\bar{\mathcal{S}}^{\mathrm{TF}}_{ab} - \frac{1}{3}\tilde{\gamma}_{ab}\mathring{D}^e h^{cd}\mathring{D}_e h_{cd},$$

How do we compute NSDs? (pt. 2)

Initial data: at a fixed ‘time’, construct a system that satisfies:

Einstein’s equations

$$\psi, \alpha, \tilde{\beta}_a, h_{ab}$$

$$\mathring{\Delta}\psi = \mathcal{S}_{\mathrm{H}}, \mathring{\Delta}\tilde{\beta}_a = \mathcal{S}_a$$

$$\mathring{\Delta}(\alpha\psi) = \mathcal{S}_{\mathrm{tr}}, \mathring{\Delta}h_{ab} = \mathcal{S}_{ab}$$

Hydrostatic equilibrium

$$\nabla_a(\rho u^\alpha) = 0, u^\alpha \nabla_\alpha s = 0$$

$$u^\beta \omega_{\beta\alpha} - T\nabla_\alpha s = 0$$

+EOS

↓

$$\frac{h}{u^t} \exp\left[\int j(\Omega) d\Omega\right] = \mathcal{E}$$

Assumptions

Axisymmetry
Stationary
Circular flow
Homentropic flow

How do we compute NSDs? (pt. 3)

1. Start with initial data for a rotating neutron star (Uryū, Tsokaros 2016)
2. Initialize a massless disk around the neutron star (Abramowicz 1977, 'Polish doughnut')

   1. the equation of state (piecewise polytrope), $P_i = K_i \rho_0^{\Gamma_i}$
   2. the rotation law, $\Omega = \eta \lambda^q$ where $\lambda := \ell / \Omega$
   3. and the location and angular momentum at the inner edge of the disk.

   Then solve hydrostatic equilibrium $\frac{h}{u^t} \exp\left[\int j(\Omega) d\Omega\right] = \mathcal{E}$

3. To add self-gravitation, iterate:
   1. first, update source terms with new contributions from disk and update gravitational potential by resolving Einstein's equations

      $\psi, \alpha, \tilde{\beta}_a, h_{ab}$

      $\mathring{\Delta}\psi = \mathcal{S}_{\mathrm{H}}, \mathring{\Delta}\tilde{\beta}_a = \mathcal{S}_a$

      $\mathring{\Delta}(\alpha\psi) = \mathcal{S}_{\mathrm{tr}}, \mathring{\Delta}h_{ab} = \mathcal{S}_{ab}$
   2. resolve hydrostatic equilibrium for both NS and disk
   3. repeat until change in variables between iterations falls below threshold

Massless disks

Range of valid disk solutions

$$\ell_{\rm in,min}/M < \ell_{\rm in}/M < \ell_{\rm in,max}/M$$

	BH	NS
χ_1	$3.532 < \ell_{\rm in}/M < 3.843$	$0.570 < \ell_{\rm in}/M < 0.636$
χ_2	$3.153 < \ell_{\rm in}/M < 3.602$	$0.455 < \ell_{\rm in}/M < 0.520$

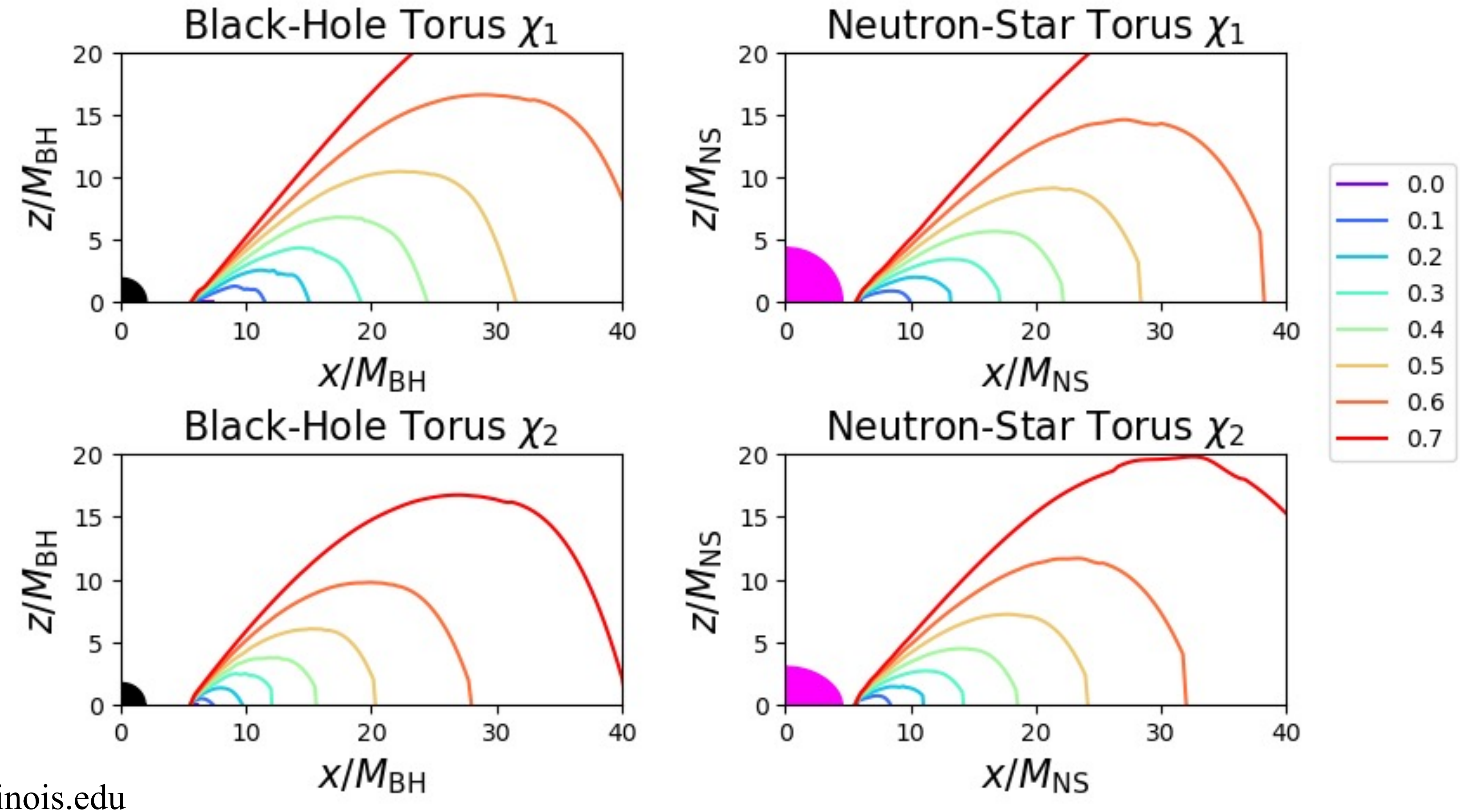
$$\chi = a/M_{\rm BH} = J_{\rm NS}/M_{\rm NS}^2$$

$$\chi_1 = 0.13$$

$$\chi_2 = 0.63$$

$$\beta = \frac{\ell_{\rm in} - \ell_{\rm in,min}}{\ell_{\rm in,max} - \ell_{\rm in,min}}$$

$$0 < \beta < 1$$

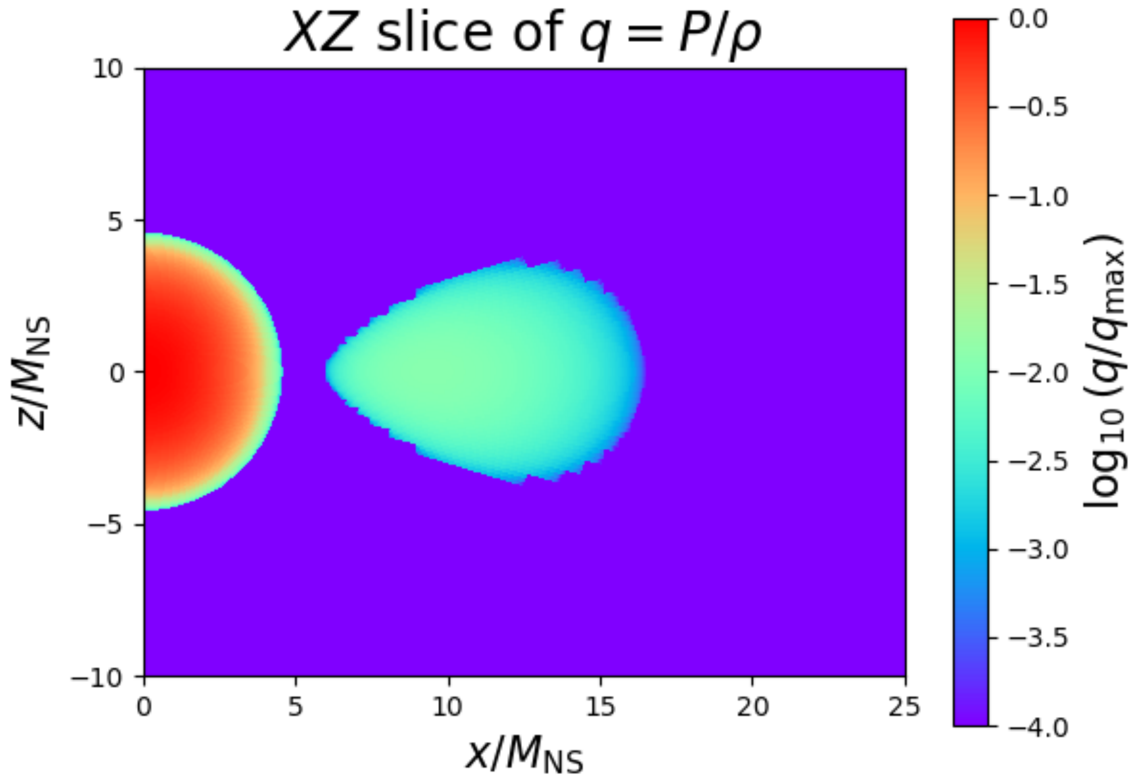


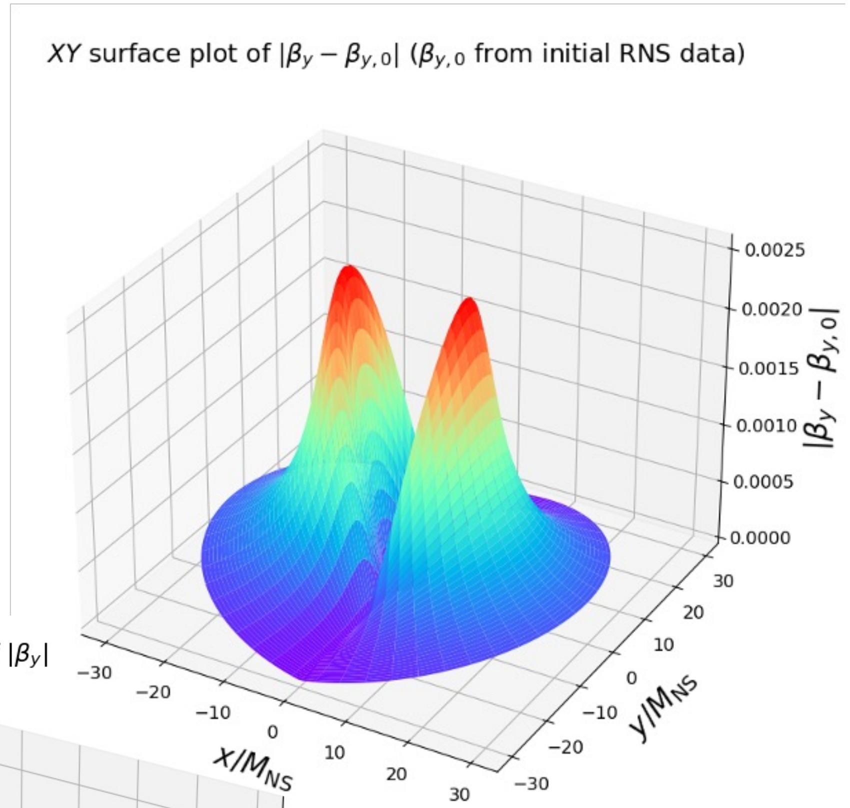
ericyu3@illinois.edu

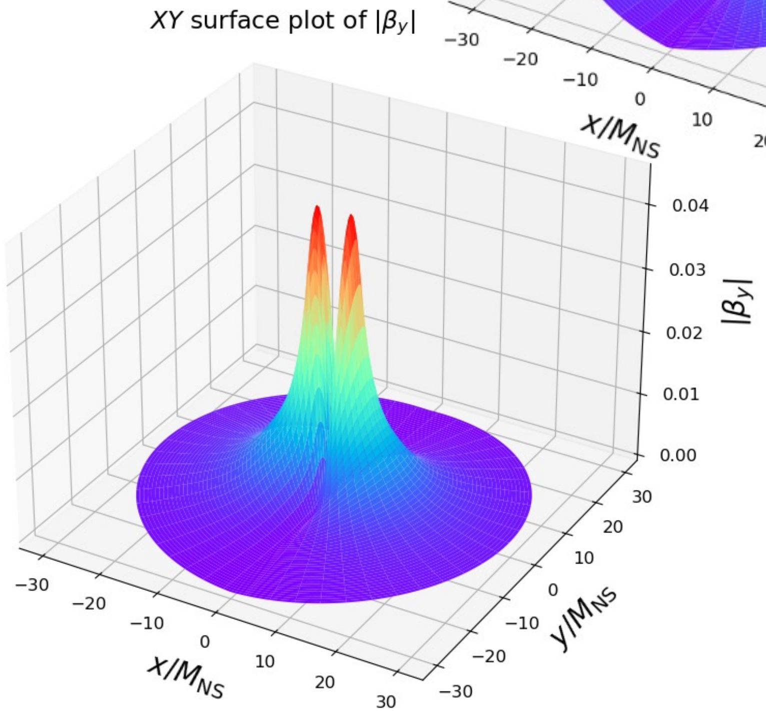
Self-gravitating disk example

$$M_{0,\mathrm{disk}} = 0.0408 M_{0,\mathrm{ns}}$$

(similar to common NSNS merger)







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